Chapter Thermal Properties of Matter



Topic-1: Thermometer & Thermal Expansion

(P)

MCQs with One Correct Answer

- 1. When a block of iron floats in mercury at 0°C, fraction k_1 of its volume is submerged, while at the temperature 60 °C, a fraction k_2 is seen to be submerged. If the coefficient of volume expansion of iron is γ_{Fe} and that of mercury is γ_{Hg} , then the ratio k_1/k_2 can be expressed as [2001S]
 - (a) $\frac{1+60\gamma_{Fe}}{1+60\gamma_{hg}}$
 - (b) $\frac{1-60\gamma_{Fe}}{1+60\gamma_{Hg}}$
 - (c) $\frac{1+60\gamma_{Fe}}{1-60\gamma_{Hg}}$
- (d) $\frac{1+60\gamma_{Hg}}{1+60\gamma_{Fe}}$
- 2. A metal ball immersed in alcohol weighs W₁ at 0°C and W₂ at 50°C. The coefficient of expansion of cubical the metal is less than that of the alcohal. Assuming that the density of the metal is large compared to that of alcohol, it can be shown that [1980]
 - (a) $W_1 > W_2$
- (b) $W_1 = W_2$
- (c) $W_1 < W_2$
- (d) None of these
- 3. A constant volume gas thermometer works on [1980]
 - (a) The Principle of Archimedes
 - (b) Boyle's Law
 - (c) Pascal's Law
 - (d) Charle's Law

(P)

2 Integer Value Answer

4. Steel wire of lenght 'L' at 40°C is suspended from the ceiling and then a mass 'm' is hung from its free end. The wire is cooled down from 40°C to 30°C to regain its original length 'L'. The coefficient of linear thermal expansion of the steel is 10⁻⁵/° C, Young's modulus of steel is 10¹¹ N/m² and radius of the wire is 1 mm. Assume that L>>diameter of the wire. Then the value of 'm' in kg is nearly [2011]

MCQs with One or More than One Correct Answer

5. A bimetallic strip is formed out of two identical strips one of copper and the other of brass. The coefficients of linear expansion of the two metals are α_c and α_B . On heating, the

temperature of the strip goes up by ΔT and the strip bends to form an arc of radius of curvature R. Then R is.

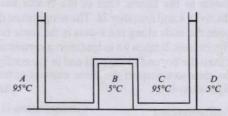
- (a) proportional to ΔT
- [1999S 3 Marks]
- (b) inversely proportional to ΔT
- (c) proportional to $|\alpha_R \alpha_C|$
- (d) inversely proportional to $|\alpha_R \alpha_C|$

O Subjective Problems

6. A cubical block of co-efficient of linear expansion α_s is submerged partially inside a liquid of co-efficient of volume expansion γ_ℓ. On increasing the temperature of the system by ΔT, the height of the cube inside the liquid remains unchanged. Find the relation between α_s and γ_ℓ.

[2004 - 4 Marks]

7. The apparatus shown in the figure consists of four glass columns connected by horizontal sections. The height of two central columns B and C are 49 cm each. The two outer columns A and D are open to the atmosphere. A and C are maintained at a temperature of 95° C while the columns B and D are maintained at 5°C. The height of the liquid in A and D measured from the base the are 52.8 cm and 51cm respectively. Determine the coefficient of thermal expansion of the liquid. [1997 - 5 Marks]



8. A sinker of weight w_0 has an apparent weight w_1 when weighed in a liquid at a temperature t_1 and w_2 when weight in the same liquid at temperature t_2 . The coefficient of cubical expansion of the material of sinker is β . What is the coefficient of volume expansion of the liquid. [1978]

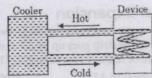


Topic-2: Calorimetry and Heat Transfer



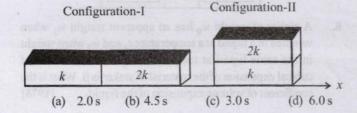
MCQs with One Correct Answer

A water cooler of storage capacity 120 litres can cool water at a constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30°C and the entire stored 120 litres of water is initially cooled to 10°C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated [Adv. 2016] for 3 hours is

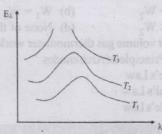


(Specific heat of water is 4.2 kJ kg⁻¹K⁻¹ and the density of water is 1000 kg m^{-3})

- (b) 2067 (a) 1600
- (c) 2533
- Parallel rays of light of intensity $I = 912 \text{ Wm}^{-2}$ are incident on a spherical black body kept in surroundings of temperature 300 K. Take Stefan-Boltzmann constant $\sigma = 5.7 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$ and assume that the energy exchange with the surroundings is only through radiation. The final steady state temperature of the black body is [Adv. 2014] close to
- (b) 660 K · (c) 990 K (d) 1550 K (a) 330 K Three rods of Copper, Brass and Steel are welded together 3. to form a Y shaped structure. Area of cross - section of each rod = 4 cm². End of copper rod is maintained at 100°C where as ends of brass and steel are kept at 0°C. Lengths of the copper, brass and steel rods are 46, 13 and 12 cms respectively. The rods are thermally insulated from surroundings excepts at ends. Thermal conductivities of copper, brass and steel are 0.92, 0.26 and 0.12 CGS units respectively. Rate of heat flow through copper rod is:
 - (a) 1.2 cal/s (c) 4.8 cal/s
- (b) 2.4 cal/s (d) 6.0 cal/s
- Two rectangular blocks, having identical dimensions, can be arranged either in configuration-I or in configuration-II as shown in the figure. One of the blocks has thermal conductivity k and the other 2k. The temperature difference between the ends along the x-axis is the same in both the configurations. It takes 9 s to transport a certain amount of heat from the hot end to the cold end in the configuration-I. The time to transport the same amount of heat in the configuration-II is [Adv. 2013]



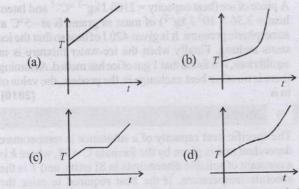
- Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is T, density of liquid is p and L is its latent heat of vaporization. [2012]
 - (a) pL/T
- (b) √T/pL
- (c) T/pL
- (d) 2T/pL
- Water of volume 2 litre in a container is heated with a coil of 1 kW at 27°C. The lid of the container is open and energy dissipates at rate of 160 J/s. In how much time temperature will rise from 27°C to 77°C [Given specific [2005 S] heat of water is 4.2 kJ/kg]
 - (c) 8 min 20 s (d) 14 min (b) 6 min 2 s
- Calorie is defined as the amount of heat required to raise temperature of 1 g of water by 1°C and it is defined under [2005 S] which of the following conditions?
 - (a) From 14.5 °C to 15.5 °C at 760 mm of Hg
 - From 98.5 °C to 99.5 °C at 760 mm of Hg
 - From 13.5 °C to 14.5 °C at 76 mm of Hg
 - (d) From 3.5 °C to 4.5 °C at 76 mm of Hg
- In which of the following process, convection does not [2005S] take place primarily
 - (a) sea and land breeze
 - (b) boiling of water
 - (c) heating air around a furnace
 - (d) warming of glass of bulb due to filament
- 9. Variation of radiant energy emitted by sun, filament of tungsten lamp and welding arc as a function of its wavelength is shown in figure. Which of the following [2005S] option is the correct match?



- (a) Sun- T_3 , tungsten filament T_1 welding arc T_2

- (b) Sun- T_2 , tungsten filament T_1 welding arc T_3 (c) Sun- T_3 , tungsten filament T_2 welding arc T_1 (d) Sun- T_1 , tungsten filament T_2 welding arc T_3
- Two identical rods are connected between two containers one of them is at 100°C and another is at 0°C. If rods are connected in parallel then the rate of melting of ice is q_1 gm/sec. If they are connected in series then the rate is q The ratio q_2/q_1 is [2004S]
 - (b) 4
- (c) 1/2
- If liquefied oxygen at 1 atmospheric pressure is heated
- from 50 k to 300 k by supplying heat at constant rate. The graph of temperature vs time will be





Three discs A, B and C having radii 2, 4, and 6 cm respectively are coated with carbon black. Wavelength for maximum intensity for the three discs are 300, 400 and 500 nm respectively. If Q_A , Q_B and Q_C are power emitted by A, B and D respectively, then

(a) Q_A will be maximum

(b) Q_B will be maximum

(c) Q_C will be maximum (d) $Q_A = Q_B = Q_C$ 2 kg of ice at -20° C is mixed with 5kg of water at 20° C in an insulating vessel having a negligible heat capacity. Calculate the final mass of water remaining in the container. It is given that the specific heats of water & ice are 1kcal/ kg/0C & 0.5 kcal/kg/0C while the latent heat of fusion of [2003S]ice is 80 kcal/kg

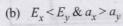
(a) 7 kg

(b) 6 kg

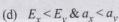
(d) 2 kg (c) 4 kg

The graph, shown in the adjacent diagram, represents the variation of temperature (T) of two bodies, x and y having same surface area, with time (t) due to the emission of radiation. Find the correct relation between the emissivity and absorptivity power of the two bodies [2003S]

(a) $E_x > E_y \& a_x < a_y$



(c) $E_x > E_y \& a_x > a_y$





- 15. An ideal Black-body at room temperature is thrown into a [2002S] furnace. It is observed that
 - initially it is the darkest body and at later times the brightest

(b) it is the darkest body at all times

it cannot be distinguished at all times (c)

initially it is the darkest body and at later times it cannot be distinguished

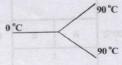
16. Three rods made of same material and having the same cross-section have been joined as shown in the figure. Each rod is of the same length. The left and right ends are kept at 0°C and 90°C respectively. The temperature of the [2001S] junction of the three rods will be

(a) 45°C

60°C (b)

(c) 30°C

(d) 20°C



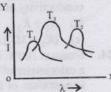
17. The plots of intensity versus wavelength for three black bodies at temperature T_1 , T_2 and T_3 respectively are as shown. Their temperatures are such that

(a) $T_1 > T_2 > T_3$

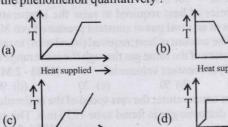
(b) $T_1 > T_2 > T_2$

(c) $T_2 > T_3 > T_1$

(d) $T_3 > T_2 > T_1$



18. A block of ice at -10°C is slowly heated and converted to steam at 100°C. Which of the following curves represents the phenomenon qualitatively?



Heat supplied -> Heat supplied → A blackbody is at a temperature of 2880 K. The energy of radiation emitted by this object with wavelength between 499 nm and 500 nm is U_1 , between 999 nm and 1000 nm is U_2 and between 1499 nm and 1500 nm is U_3 . The Wien constant $b = 2.88 \times 10^6$ nm K. Then [1998S - 2 Marks]

(a) $U_1 = 0$ (b) $U_3 = 0$ (c) $U_1 > U_2$ (d) $U_2 > U_1$

A spherical black body with a radius of 12 cm radiates 450 W power at 500 K. if the radius were halved and the temperature doubled, the power radiated in watt would be [1997 - 1 Mark]

(c) 900 (b) 450 Two metallic spheres S_1 and S_2 are made of the same 21. material and have got identical surface finish. The mass of S_1 is thrice that of S_2 . Both the spheres are heated to the same high temperature and placed in the same room having lower temperature but are thermally insulated from each other. The ratio of the initial rate of cooling of S_1 to that of [1995S]

Three rods of identical cross-sectional area and made from the same metal from the sides of an isosceles traingle ABC, right-angled at B. The points A and B are maintained at

> temperatures T and $(\sqrt{2})$ T respectively. In the steady state, the temperature of the point C is T_c . Assuming that only heat conduction takes place, T_c/T is [19958]

(a)
$$\frac{1}{2(\sqrt{2}-1)}$$
 (b) $\frac{3}{\sqrt{2}+1}$ (c) $\frac{1}{\sqrt{3}(\sqrt{2}-1)}$ (d) $\frac{1}{\sqrt{2}+1}$

23. A cylinder of radius R made of a material of thermal conductivity K_1 is surrounded by a cylindrical shell of inner radius R and outer radius 2R made of a material of thermal conductivity K_2 . The two ends of the combined

system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is in steady state. The effective thermal conductivity of the system is [1988-2 Marks]

(a) $K_1 + K_2$

(b) $K_1K_2/(K_1+K_2)$ (d) $(3K_1+3K_2)/4$

(c) (K₁+3K₂)/4 (d) (3K₁+3K₂)/4
 24. Steam at 100°C is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15°C till the temperature of the calorimeter and its contents rises to 80°C. The mass of the steam condensed in kilogram is

[1986 - 2 Marks]

(d) 0.135

(a) 0.130 (b) 0.065 (c) 0.260

25. 70 calories of heat required to raise the temperature of 2 moles of an ideal gas at constant pressure from 30°C to 35°C. The amount of heat required (in calories) to raise the temperature of the same gas through the same range (30°C to 35°C) at constant volume is: [1985 - 2 Marks]
(a) 30 (b) 50 (c) 70 (d) 90

26. At room temperature, the rms speed of the molecules of a certain diatomic gas is found to be 1930 m/s. The gas is

(a) H_2 (b) F_2 (c) O_2 (d) Cl_2

A wall has two layers A and B, each made of different material. Both the layers have the same thickness. The thermal conductivity of the meterial of A is twice that of B. Under thermal equilibrium, the temperature difference across the wall is 36°C. The temperature difference across the layer A is [1980]
(a) 6°C
(b) 12°C
(c) 18°C
(d) 24°C

2 Integer/Non-negative Integer Value Answer

28. A small object is placed at the center of a large evacuated hollow spherical container. Assume that the container is maintained at 0 K. At time t = 0, the temperature of the object is 200 K. The temperature of the object becomes 100 K at $t = t_1$ and 50 K at $t = t_2$. Assume the object and the container to be ideal black bodies. The heat capacity of the object does not depend on temperature. The ratio (t_2/t_1) is ______. [Adv. 2021]

29. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays log₂, (P/P₀), where P₀ is a constant. When the metal surface is at a temperature of 487°C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767°C?

[Adv. 2016]

30. Two spherical stars A and B emit blackbody radiation. The radius of A is 400 times that of B and A emits 10^4 times the

power emitted from B. The ratio $\left(\frac{\lambda_A}{\lambda_B}\right)$ of their wavelengths λ_A and λ_B at which the peaks occur in their respective

radiation curves is [Adv. 2015]
31. Two spherical bodies A (radius 6 cm) and B(radius 18 cm) are at temperature T_1 and T_2 , respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B? [2010]

32. A piece of ice (heat capacity = $2100 \text{ J kg}^{-1} \, \text{°C}^{-1}$ and latent heat = $3.36 \times 10^5 \, \text{J kg}^{-1}$) of mass m grams is at $-5 \, \text{°C}$ at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally when the ice-water mixture is in equilibrium, it is found that 1 gm of ice has melted. Assuming there is no other heat exchange in the process, the value of m is [2010]

3 Numeric Answer

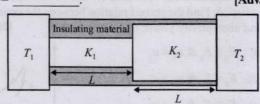
33. The specific heat capacity of a substance is temperature dependent and is given by the formula C = kT, where k is a constant of suitable dimensions in SI units, and T is the absolute temperature. If the heat required to raise the temperature of 1 kg of the substance from -73° C to 27° C is nk, the value of n is _____. [Given: $0 \text{ K} = -273^{\circ}$ C.]

[Adv. 2024]

34. A liquid at 30°C is poured very slowly into a Calorimeter that is at temperature of 110°C. The boiling temperature of the liquid is 80°C. It is found that the first 5gm of the liquid completely evaporates. After pouring another 80gm of the liquid to its specific heat will be ___ °C.

[Neglect the heat exchange with surrounding] [Adv. 2019]

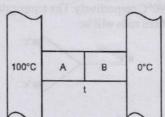
75. Two conducting cylinders of equal length but different radii are connected in series between two heat baths kept at temperatures $T_1 = 300 \text{ K}$ and $T_2 = 100 \text{ K}$, as shown in the figure. The radius of the bigger cylinder is twice that of the smaller one and the thermal conductivities of the materials of the smaller and the larger cylinders are K_1 and K_2 respectively. If the temperature at the junction of the two cylinders in the steady state is 200 K, then $K_1/K_2 = \frac{1}{100 \text{ K}}$.



Fill in the Blanks

36. Earth receives 1400 W/m^2 of solar power. If all the solar energy falling on a lens of area 0.2 m^2 is focused on to a block of ice of mass 280 grams, the time taken to melt the ice will be... minutes. (Latent heat of fusion of ice = $3.3 \times 10^5 \text{ J/kg.}$) [1997 - 2 Marks]

37. Two metal cubes A and B of same size are arranged as shown in Figure. The extreme ends of the combination are maintained at the indicated temperatures. The arrangement is thermally insulated. The coefficients of thermal conductivity of A and B are 300 W/m °C and 200 W/m °C, respectively. After steady state is reached the temperature t of the interface will be [1996 - 2 Marks]

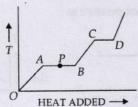




- 38. A substance of mass M kg requires a power input of P watts to remain in the molten state at its melting point. When the power source is turned off, the sample completely solidifies in time t seconds. The latent heat of fusion of the [1992 - 1 Mark] substance is
- A point source of heat of power P is placed at the centre of a spherical shell of mean radius R. The material of the shell has thermal conductivity K. If the temperature difference between the outer and inner surface of the shell is not to exceed T, the thickness of the shell should not be less than [1991 - 1 Mark]
- A solid copper sphere (density ρ and specific heat c) of radius r at an initial temperature 200 K is suspended inside a chamber whose walls are at almost 0K. The time required for the temperature of the sphere to drop to 100K is [1991 - 2 Marks]
- The earth receives at its surface radiation from the sun at the rate of 1400 W/m⁻². The distance of the centre of the sun from the surface of the earth is 1.5×10^{11} m and the radius of the sun is 7×10^8 m. Treating the sun as a black body, it follows from the above data that its surface [1989 - 2 Marks] temperature is.....K.
- 300 grams of water at 25° C is added to 100 grams of ice at 0°C. The final temperature of the mixture is°C.

[1989 - 2 Marks]

The variation of temperature of a material as heat is given to it at a constant rate is shown in the figure. The material is in solid state at the point O. The state of the material at [1985 - 2 Marks] the point P is

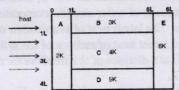


True / False

Two spheres of the same meterial have radii 1 m and 4 m and temperatures 4000K and 2000K respectively. The energy radiated per second by the first sphere is greater [1988 - 2 Marks] than that by the second.

6 MCQs with One or More than One Correct Answer

A composite block is made of slabs A, B, C, D and E of different thermal conductivities (given in terms of a constant K and sizes (given in terms of length, L) as shown in the figure. All slabs are of same width. Heat 'Q' flows only from left to right through the blocks. Then in steady [2011] state



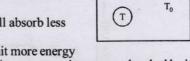
- (a) heat flow through A and E slabs are same.
- (b) heat flow through slab E is maximum.

- (c) temperature difference across slab E is smallest.
- heat flow through C = heat flow through B + heat flow through D.
- A black body of temperature T is inside chamber of T_0 temperature initially. Sun rays are allowed to fall from a hole in the top of chamber. If the temperature of black body (T) and chamber (T_0) remains constant, then

[2006 - 5M, -1]

- Black body will absorb more radiation
- Black body will absorb less radiation

Black body emit more energy



- (d) Black body emit energy equal to energy absorbed by it Two bodies A and B have thermal emissivities of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are the same. The two bodies emit total radiant power of the same rate. The wavelength λ_B corresponding to maximum spectral radiancy in the radiation from B shifted from the wavelenth corresponding to maximum spectral radiancy in the radiation from A, by 1.00 µm. If the temperature of A is 5802 K: [1994 - 2 Marks]
 - (a) the temperature of B is 1934 K
 - $\lambda_B = 1.5 \, \mu \text{m}$
 - the temperature of B is 11604 K
 - the temperature of B is 2901 K

Subjective Problems

- 0.05 kg steam at 373 K and 0.45 kg of ice at 253K are mixed in an insulated vessel. Find the equilibrium temperature of the mixture. Given, $L_{fusion} = 80 \text{ cal/g} = 336 \text{ J/g}, L_{vaporization} = 540 \text{ cal/g} = 2268 \text{ J/g}, S_{ice} = 2100 \text{ J/Kg K} = 0.5 \text{ cal/gK and } S_{water} = 4200 \text{ J/Kg K} = 1 \text{ cal/gK}$ [2006 - 6M]
- 49. An ice cube of mass 0.1 kg at 0°C is placed in an isolated container which is at 227°C. The specific heat S of the container varies with temperature T according to the empirical relation S = A + BT, where A = 100 cal/kg-K and B $= 2 \times 10^{-2}$ cal/kg- K^2 . If the final temperature of the container is 27°C, determine the mass of the container. (Latent heat of fusion of water = 8×10^4 cal/kg, Specific heat of water = 103 cal/kg-K). [2001-5 Marks]
- The temperature of 100g of water is to be raised from 24°C to 90°C by adding steam to it. Calculate the mass of the [1996 - 2 Marks] steam required for this purpose.
- A solid sphere of copper of radius R and a hollow sphere of the same material of inner radius r and outer radius A are heated to the same temperature and allowed to cool in the same environment. Which of them starts cooling faster? [1982 - 2 Marks]
 - A lead bullet just melts when stopped by an obstacle. Assuming that 25 per cent of the heat is absorbed by the

obstacle, find the velocity of the bullet if its initial

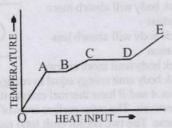
temperature is 27°C.



(Melting point of lead = 327°C, specific heat of lead = 0.03 calories/gm/°C, latent heat of fusion of lead = 6 calories/gm, J = 4.2 joules/calorie). [1981-3 Marks]

53. A Solid material is supplied with heat at a constant rate.

The temperature of the material is changing with the heat input as shown in the graph in figure. Study the graph carefully and answer the following questions: [1980]

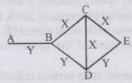


(i) What do the horizontal regions AB and CD represent?

(ii) If CD is equal to 2AB, what do you infer?

(iii) What does the slope of *DE* represent?

- (iv) The slope of OA > the slope of BC. What does this indicate?
- 54. Three rods of material X and three rods of material Y are connected as shown in the figure. All the rods are of identical length and cross-sectional area. If the end A is maintained at 60°C and the junction E at 10°C. Calculate the temperature of the junctions B, C and D. The thermal conductivity of X is 0.92 cal/sec-cm-°C and that of Y is 0.46 cal/sec-cm-°C.





Topic-3: Miscellaneous (Mixed Concepts) Problems



MCQs with One Correct Answer

1. A current carrying wire heats a metal rod. The wire provides a constant power P to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature (T) is the metal rod changes with time (t) as $T(t) = T_0(1+\beta t^{2/4})$

Where β is a constant with appropriate dimensions while T_0 is a constant with dimensions of temperature. The heat capacity of metal is: [Adv. 2019]

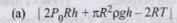
(a)
$$\frac{4P(T(t) - T_0)}{\beta^4 T_0^2}$$

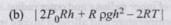
(b)
$$\frac{4P(T(t)-T_0)^2}{\beta^4 T_0^3}$$

(c)
$$\frac{4P(T(t)-T_0)^4}{\beta^4 T_0^5}$$

(d)
$$\frac{4P(T(t)-T_0)^3}{\beta^4 T_0^4}$$

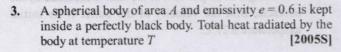
2. Water is filled up to a height h in a beaker of radius R as shown in the figure. The density of water is ρ, the surface tension of water is T and the atmospheric pressure is P₀. Consider a vertical section ABCD of the water column through a diameter of the beaker. The force on water on one side of this section by water on the other side of this section has magnitude [2007]





(c)
$$|P_0\pi R^2 + R\rho g h^2 - 2RT|$$

(d)
$$|P_0\pi R^2 + R\rho gh^2 + 2RT|$$



(a)
$$0.4AT^4$$

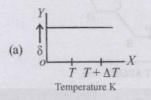
(c)
$$0.6AT^4$$

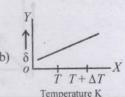
4. An ideal gas is initially at temperature T and volume V. Its volume is increased by ΔV due to an increase in temperature

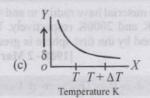
 ΔT , pressure remaining constant. The quantity $\delta = \frac{\Delta V}{V \Delta T}$

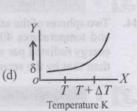
varies with temperature as











(2)

2 Integer Value Answer

5. A metal rod AB of length 10x has its one end A in ice at 0. °C, and the other end B in water at 100 °C. If a point P onthe rod is maintained at 400 °C, then it is found that equal amounts of water and ice evaporate and melt per unit time. The latent heat of evaporation of water is 540 cal/g and latent heat of melting of ice is 80 cal/g. If the point P is at a distance of λx from the ice end A, find the value of λ

[Neglect any heat loss to the surrounding.]

[2009]





3 Numeric Answer

- 6. A container with 1 kg of water in it is kept in sunlight, which causes the water to get warmer than the surroundings. The average energy per unit time per unit area received due to the sunlight is 700 Wm⁻² and it is absorbed by the water over an effective area of 0.05 m². Assuming that the heat loss from the water to the surroundings is governed by Newton's law of cooling, the difference (in °C) in the temperature of water and the surroundings after a long time will be ______. (Ignore effect of the container, and take constant for Newton's law of cooling = 0.001 s⁻¹, Heat capacity of water = 4200 J kg⁻¹ K⁻¹) [Adv. 2020]
- (:Q:

MCQs with One or More than One Correct Answer

7. A human body has a surface area of approximately 1 m^2 . The normal body temperature is 10 K above the surrounding room temperature T_0 . Take the room temperature to be $T_0 = 300 \text{ K}$. For $T_0 = 300 \text{ K}$, the value of

 $\sigma T_0^4 = 460 \text{ Wm}^{-2}$ (where σ is the Stefan-Boltzmann constant). Which of the following options is/are correct? [Adv. 2017]

- (a) The amount of energy radiated by the body in 1 second is close to 60 joules
- (b) If the surrounding temperature reduces by a small amount $\Delta T_0 \ll T_0$, then to maintain the same body temperature the same (living) human being needs to radiate $\Delta W = 4\sigma T_0^3 \Delta T_0$ more energy per unit time
- (c) Reducing the exposed surface area of the body (e.g. by curling up) allows humans to maintain the same body temperature while reducing the energy lost by radiation
- (d) If the body temperature rises significantly then the peak in the spectrum of electromagnetic radiation emitted by the body would shift to longer wavelengths



Answer Key

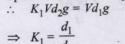
| | | | | | Topic-1 : Thermometer & Thermal Expansion | | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|---|------|--------|--------|---------|-----|---------|------|---------|-----|-----|-----|--------|-----|-----|
| 1. | (a) | 2. | (c) | 3. | (d) | 4. | (3) | 5. | (b, d) | | | | | | | | | | |
| | | | | | То | pic- | 2 : Co | lorin | netry o | and | Heat T | ran | sfer | | | | | | |
| 1. | (b) | 2. | (a) | 3. | (c) | 4. | (a) | 5. | (d) | 6. | (c) | 7. | (a) | 8. | (d) | 9. | (a) | 10. | (d) |
| 11. | (c) | 12. | (b) | 13. | (b) | 14. | (c) | 15. | (a) | 16. | (b) | 17. | (b) | 18. | (a) | 19. | (d) | 20. | (d) |
| 21. | (d) | 22. | (b) | 23. | (c) | 24. | (a) | 25. | (b) | 26. | (a) | 27. | (b) | 28. | (9) | 29. | (9) | 30. | (2) |
| 31. | (9) | 32. | (8) | 33. | (25000) | 34. | (270 | °C)35. | (4.00) | 44. | (False) | 45. | (a,c,d) | 46. | (d) | 47. | (a, b) | | |
| | | | | | Topic-3 | : Mi | scelle | aneo | us (Mi | ked | Concep | ots) | Proble | ms | | | | | |
| 1. | (d) | 2. | (b) | 3. | (d) | 4. | (c) | 5. | (9) | 6. | (8.33) | 7. | (c) | | | | | | |

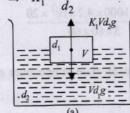
Hints & Solutions

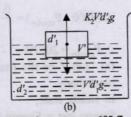


Topic-1: Thermometer & Thermal Expansion

(a) For equilibrium when temperature 0°C fig (a) Upthrust = Wt. of body







... (i)

For equilibrium when temperature increases to 60° fig. (b) When the temperature is increased the density will

$$\therefore d_1' = d_1 (1 + \gamma_{Fe} \times 60)$$
and $d_2' = d_2 (1 + \gamma_{Hg} \times 60)$
Again upthrust = Wt. of body

$$K_2Vd_2'g = Vd_1'g$$

$$\therefore K_2 \left[\frac{d_2}{1 + \gamma_{Hg} \times 60} \right] = \frac{d_1}{1 + \gamma_{Fe} \times 60}$$

$$K_2 \left[\frac{1 + \gamma_{Fe} \times 60}{1 + \gamma_{Hg} \times 60} \right] = \frac{d_1}{d_2} \Rightarrow \frac{K_1}{K_2} = \frac{1 + \gamma_{Fe} \times 60}{1 + \gamma_{Hg} \times 60}$$

(c)
$$W_1 = mg - Va_a g$$

 $W_2 = mg - V'd'_a g = mg - V(1 + 50 \gamma_b) \frac{d_a g}{(1 + 50 \gamma_a)}$

$$= mg - V d_a g \left[\frac{1 + 50 \gamma_b}{1 + 50 \gamma_a} \right]$$

$$1 + 50 \gamma_b < 1 + 50 \gamma_a \quad \text{or,} \quad \frac{1 + 50 \gamma_b}{1 + 50 \gamma_a} < 1$$

- $W_2 > W_1 \text{ or } W_1 < W_2$
- (d) As constant volume gas thermometer, works on
- 4. (3) We know that, $Y = \frac{F}{4} / \frac{\Delta l}{l}$

$$\therefore Y = \frac{mg/A}{\Delta\ell/\ell} = \frac{mg\ell}{A\Delta\ell} \qquad \dots (i)$$

Also
$$\Delta \ell = \ell \alpha \Delta T$$
 ...(i

From (i) and (ii)

$$Y = \frac{mg\ell}{4\ell \alpha \Lambda T} = \frac{mg}{4\alpha \Lambda T}$$

$$\therefore m = \frac{YA \alpha \Delta T}{g} = \frac{10^{11} \times \pi (10^{-3})^2 \times 10^{-5} \times 10}{10} = \pi \approx 3$$

(b, d) Let L_0 be the original length of the strip. Co-efficient of linear expansion of brass is greater than

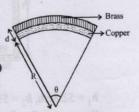
that of copper i.e., $\alpha_B > \alpha_C$.

$$L_B = L_0(1 + \alpha_B \Delta T) (R + d)\theta$$

= $L_0(1 + \alpha_B \Delta T)$

Again,
$$L_C = L_0(1 + \alpha_C \Delta T) = R\theta$$

$$\therefore \frac{(R+d)\theta}{R\theta} = \frac{1+\alpha_B \Delta T}{1+\alpha_C \Delta T}$$



or,
$$\frac{R+d}{R} = (1 + \alpha_B \Delta T) (1 - \alpha_C \Delta T)$$
,

[By binomial expansion]

or,
$$1 + \frac{d}{R} = 1 + (\alpha_B - \alpha_C)\Delta T - \alpha_B \alpha_C (\Delta T)^2$$

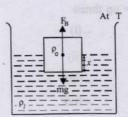
or,
$$\frac{d}{R} = (\alpha_B - \alpha_C) \Delta T$$
 or $R = \frac{d}{(\alpha_B - \alpha_C) \Delta T}$

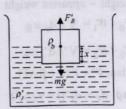
$$\therefore R \propto \frac{1}{\Delta T} \text{ and } R \propto \frac{1}{|\alpha_B - \alpha_C|}.$$

Initially, at temperature T bnoyant force

$$F_B = \text{mg or, } Ax\rho_\ell g = AL\rho_b g$$

$$\therefore x \rho_{\ell} = L \rho_{b} \qquad \dots (i$$





At temperature $T + \Delta T$ the volume of the cube increases but the density of liquid decreases so depth upto which the cube is immersed in the liquid remains same.

$$F_B' = mg$$

or,
$$A'x\rho'_{\ell}g = AL\rho_b g$$

100°C

Copper

Now,
$$A' = A (1 + 2\alpha \Delta T)$$

$$\rho'_{\ell} = \rho_{\ell} (1 - \gamma \Delta T)$$

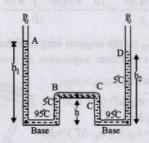
$$\therefore A(1+2\alpha \Delta T) \times \rho_{\ell}(1-\gamma \Delta T)g = AL\rho_{h}g$$

$$\Rightarrow x\rho_{\ell}(1+2\alpha\Delta T)(1-\gamma\Delta T)=L\rho_{h}$$

$$\Rightarrow x\rho_{\ell}(1+2\alpha \Delta T)(1-\gamma \Delta T) = x\rho_{\ell}$$
 [from eq. (i)]

$$\Rightarrow$$
 1 + 2\alpha \Delta T - \gamma \Delta T = 1 \Rightarrow \gamma = 2\alpha \text{ or, } \gamma_I = 2\alpha \text{s}

Density of a liquid decreases as temperature rises according to relatoin $d_0 = d_t (1 + \gamma t)$ where $\gamma = \text{co-efficient of cubical}$ expansion of liquid.



$$h_1 = 52.8 \text{cm}, h_2 = 51 \text{ cm}, h = 49 \text{ cm}$$

Pressure at
$$B =$$
Pressure at C

$$\begin{split} P_0 &= d_{95^{\circ}\text{C}} \, g h_1 - d_{5^{\circ}\text{C}} \, g h = P_0 + d_{5^{\circ}\text{C}} \, g h_2 - d_{95^{\circ}\text{C}} \, g h \\ \text{or} \quad d_{95^{\circ}\text{C}} \, (h_1 + h) &= d_{5^{\circ}\text{C}} \, (h_2 + h) \end{split}$$

or
$$\frac{d_{95^{\circ}C}}{d_{5^{\circ}C}} = \frac{h_2 + h}{h_1 + h} \text{ or } \frac{d_0 / (1 + 95\gamma)}{d_0 / (1 + 5)} = \frac{h_2 + h}{h_1 + h}$$

or
$$\frac{1+5\gamma}{1+95\gamma} = \frac{51+49}{52.8+49} = \frac{100}{101.8} = \frac{1}{1.018}$$

or
$$1+95\gamma=1.018+5\times1.018\gamma$$

or
$$95\gamma - 5.090\gamma = 0.018$$
 or $\gamma = \frac{0.018}{89.91} \times 2 \times 10^{-4}$

$$\therefore \quad \text{Coefficient of linear expansion } \alpha = \frac{\gamma}{3} = \frac{2 \times 10^{-4}}{3}$$

$$\alpha = 6.67 \times 10^{-5} \text{ per }^{\circ}\text{C}.$$

Let y, be the coefficient of volume expansion of actual weight - apparent weight = up thrust

$$W_0 - W_1 = V \times d_{\ell} \times g$$

$$W_0 - W_2 = V' \times d'_{\ell} \times g$$
 ... (ii)

Also,
$$V' = V(1 + \beta \Delta T)$$
 ... (iii)

and
$$d_{\ell} = d_{\ell}' (1 + \gamma_{\ell} \Delta T)$$
 ... (iv)

From eq. (ii), (iii) and (iv)

$$W_0 - W_2 = \frac{V(1 + \beta \Delta T) \times d_{\ell}}{1 + \gamma_{\ell} \Delta T} \times g \quad \dots (v)$$

Dividing (i) by (v), we get

$$\frac{W_0 - W_1}{W_0 - W_2} = \frac{V d_{\ell} g \left(1 + \gamma_{\ell} \Delta T\right)}{V \left(1 + \beta \Delta T\right) d_{\ell} g}$$

$$\Rightarrow \frac{W_0 - W_1}{W_0 - W_2} = \frac{1 + \gamma_{\ell} \Delta T}{1 + \beta \Delta T} \Rightarrow \frac{W_0 - W_1}{W_0 - W_2} = \frac{1 + \gamma_{\ell} (t_2 - t_1)}{1 + \beta (t_2 - t_1)}$$

$$\Rightarrow (W_0 - W_1) [1 + \beta (t_2 - t_1) = (W_0 - W_2) [1 + \gamma_{\ell} (t_2 - t_1)]$$

$$\Rightarrow \quad \gamma_{\ell} = \frac{W_2 - W_1}{(W_0 - W_2)(t_2 - t_1)} + \frac{\beta(W_0 - W_1)}{(W_0 - W_2)}$$



Topic-2: Calorimetry and Heat Transfer

1. **(b)**
$$P_{\text{heater}} - P_{\text{cooler}} = \frac{mc\Delta T}{t} = \frac{V \rho c \Delta T}{t}$$

$$\therefore (3000 - P_{cooler}) = \frac{0.12 \times 1000 \times 4.2 \times 10^3 \times 20}{3 \times 60 \times 60}$$

:. $P_{cooler} = 2067W$ (a) Let T be the final steady state temperature of the black body.

$$\sigma(T^4 - T_0^4) \times 4\pi R^2 = I(\pi R^2)$$

$$\therefore 5.7 \times 10^{-8} \left[T^4 - (300)^4 \right] \times 4 = 912 \therefore T = 330 \text{ K}$$

(c) Rate of heat flow is given by,

$$Q = \frac{KA(\theta_1 - \theta_2)}{l}$$

Where, K = coefficient of thermal

$$l = \text{length of rod and A} = \text{area of } 0^{\circ}\text{C}$$

cross-section of rod

If the junction temperature is T, then

$$Q_{Copper} = Q_{Brass} + Q_{Steel}$$

$$\frac{0.92 \times 4(100 - T)}{46} = \frac{0.26 \times 4 \times (T - 0)}{13} + \frac{0.12 \times 4 \times (T - 0)}{12}$$

$$\Rightarrow$$
 200-2T=2T+T \Rightarrow T=40°C

$$\therefore Q_{\text{Copper}} = \frac{0.92 \times 4 \times 60}{46} = 4.8 \text{ cal/s}$$

(a) Equivalent thermal resistance in configuration-I

$$R_3 = R_1 + R_2 = \frac{L}{KA} + \frac{L}{2KA} = \frac{3}{2} \frac{L}{KA}$$

Equivalent thermal resistance in configuration-II

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{KA}{l} + \frac{2KA}{l}$$
 or, $R_P = \frac{l}{3KA} = \frac{R_s}{4.5}$

i.e., Thermal resistance in configuration-II, R_P is 4.5 times less than thermal resistance in configuration - I, R,

$$\therefore \quad 4.5t_P = t_s \implies t_P = \frac{t_s}{4.5} = \frac{9}{4.5}s = 2s$$

(d) When radius is decrease by ΔR , Decrease in surface energy = Heat lost as latent heat

$$(4\pi R^2 \Delta R \rho) L = 4\pi [R^2 - (R - \Delta R)^2] T$$

$$\Rightarrow \rho R^2 \Delta RL = T[R^2 - R^2 + 2R\Delta R - \Delta R^2]$$

$$\Rightarrow$$
 ρR²ΔRL = T2RΔR [ΔR is very small]

$$\Rightarrow R = \frac{2T}{\rho L}$$

(c) As shown in the figure, the net heat gained by the water to raise its temperature

=(1000-160)=840 J/s

Now, the heat required to raise the temperature of water from 27° C to 77°C

$$Q = mc \Delta t = 2 \times 4200 \times 50 \text{ J}$$

Hence time required to gain Q amount of heat



$$t = \frac{Q}{840} = \frac{2 \times 4200 \times 50}{840} = 500 \text{ s} = 8 \text{ min } 20 \text{ s}$$

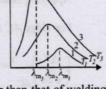
- (a) 1 Calorie is the amount of heat required to raise temperature of 1 gram of water from 14.5°C to 15.5°C at 760 mm of Hg.
- (d) Warming of glass of bulb due to filament is primarily 8. due to radiation. A medium is required for convection process. As a bulb is almost evacuated, heat from the filament is transmitted through radiation.
- (a) According to Wein's displacement law

$$\lambda_m \times T = \text{constant}$$

From the graph, $\lambda_{m_3} < \lambda_{m_2} < \lambda_{m_1}$

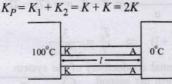
From the graph,
$$n_{m_3} < n_{m_2} < T_1$$

$$T_3 > T_2 > T_1$$



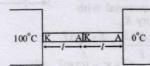
The temperature of $Sun - T_3$ is higher than that of welding arc T_2 which in turn is greater than tungsten filament $-T_1$.

10. (d) In parallel combination of rods



In series combination

$$K_S = \frac{K_1 - K_2}{K_1 + K_2} = \frac{KK}{2K} = \frac{K}{2}$$



$$q_1 = \frac{2KA(100)}{\ell}$$
 and $q_2 = \frac{KA(100)}{2}$

$$\therefore \frac{q_2}{q_1} = \frac{KA(100)}{2\ell} \times \frac{\ell}{2KA(100)} = \frac{1}{4}$$

11. (c) From 50 K to boiling temperature, T increases linearly as $Q = mc \Delta T$. Hence T - t graph will be a straight line inclined to time axis.

During boiling,
$$Q = mL$$

Temperature remains constant till boiling is complete and graph will be a straight line paralls to time axis.

After that, temperature increases linearly T-t graph will be a straight line inclined to time axis.

(b) From wein's displacement law

$$\lambda_{\rm m}T = {\rm Constant}$$

$$T = \frac{\text{constant}}{\lambda m}$$

$$\left\{ :: T_A = \frac{C}{3 \times 10^{-7}}, T_B = \frac{C}{4 \times 10^{-7}}, T_C = \frac{C}{5 \times 10^{-7}} \right\}$$

Again from stefan's law

$$Q = \sigma A T^4$$
 : $Q_A = \sigma . \pi (2 \times 10^{-2})^2 \times \frac{C^4}{81 \times 10^{-28}}$

$$Q_B = \sigma.\pi (4 \times 10^{-2})^2 \times \frac{C^4}{256 \times 10^{-28}}$$

and
$$Q_C = \sigma.\pi (6 \times 10^{-2})^2 \times \frac{C^4}{625 \times 10^{-28}}$$

Hence
$$Q_B > Q_C > Q_A$$

(b) Heat released when 5kg of water at 20°C falls to 0°C temperature = $mC_{\infty} \Delta T = 5 \times 1 \times 20 = 100 \text{ kcal}$

Heat required by 2 kg ice at - 20°C to convert into 2 kg of ice at 0° C = $mC_{ice} \Delta T = 2 \times 0.5 \times 20 = 20 \text{ k cal.}$

Hence remaining 100 - 20 = 8 kcal of heat will melt m kg ice at 0°C to water at 0°C

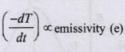
$$Q = mL \Rightarrow 80 = m \times 80 \Rightarrow m = 1 \text{ kg}$$

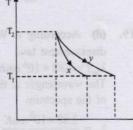
Therefore the amount of water at 0° C = 5 kg + 1kg = 6 kg And amount of ice at $0^{\circ}C = 2 - 1 = 1$ kg.

14. (c) From the graph,

$$\left(\frac{-dT}{dt}\right)_x > \left(-\frac{dT}{dt}\right)_y$$

Rate of cooling,





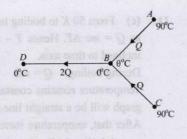
$$\therefore E_x > E_y$$

Also $a_x > a_y$ as good absorbers are good emitters.

(a) According to Kirchoff's law, good absorbers are good emitters and bad reflectors.

At high temperature (in the furnace), since it absorbs more energy, it emits more radiations as well and hence is the brightest and it is the darkest body initially.

(b) Let θ°C be the temperature of junction at B. Let Q is the heat flowing per second from A at $90^{\circ}C$ to B at $\theta^{\circ}C$ on account of



temperature difference.
$$\therefore Q = \frac{KA(90 - \theta)}{\ell} \dots (i)$$

$$Q = \frac{KA(90 - \theta)}{l}$$

 \therefore The heat flowing per second from B to D

$$2Q = \frac{KA(\theta - 0)}{\ell} \qquad \dots (ii)$$

Dividing eq. (ii) by (i)

$$2 = \frac{\theta}{90 - \theta} \implies \theta = 60^{\circ}$$

Hence temperature of the junction $\theta = 60^{\circ}\text{C}$

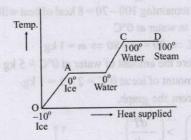
- **(b)** According to Wien's displacement law, $\lambda T = \text{constant}$ 17. From graph $\lambda_1 < \lambda_3 < \lambda_2$: $T_1 > T_3 > T_2$.
- 18.

 $O \rightarrow A$, the temperature of ice changes from -10° C to 0° C.

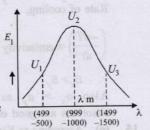
 $A \rightarrow B$, ice at 0°C melts into water at 0°C.

 $B \rightarrow C$, water at 0°C changes into water at 100°C.

 $C \rightarrow D$, water at 100°C changes into steam at 100°C.



19. (d) According to Wien's displacement law, $\lambda_m T = 2.88 \times 10^6 \, nmK$ The wavelength at the peak



 $\lambda_m = \frac{2.88 \times 10^6 \, nmK}{2880 \, K} = 10^3 \, nm$

It follows that the energy radiated between 499 nm to 500 nm will be less than that emitted between 999 nm to 1000 nm, i.e., $U_1 < U_2$ or $U_2 > U_1$.

(d) From Stefan's law energy radiated per second by a black body

 $\frac{E}{t} = \sigma T^4 \times A = \sigma T^4 \times 4\pi r^2$ $\frac{E}{t}$ = 450 when T = 500 K, r = 0.12 m $450 = 4\pi\sigma (500)^4 (0.12)^2$

when $T = 1000 \, K$, $r = 0.06 \, \text{m}$, $\frac{E}{r} = ?$

$$\therefore \frac{E}{t} = 4\pi\sigma (1000)^4 (0.06)^2 \qquad ... (ii)$$

Dividing eq. (ii) by (i), we get

$$\frac{E/t}{450} = \frac{(1000)^4 (0.06)^2}{(500)^4 (0.12)^2} = \frac{2^4}{2^2} = 4 \text{ or, } \frac{E}{t} = 450 \times 4 = 1800 \text{ W}$$
21. (d) According to Stefan's law, $\Delta Q = e \sigma A T^4 \Delta t$

Also, $\Delta Q = mc \Delta T$

or,
$$mc \Delta T = e\sigma A T^4 \Delta t$$

or,
$$\frac{\Delta T}{\Delta t} = \frac{e\sigma A T^4}{mc} = \frac{e\sigma T^4}{mc} \left[\pi \left(\frac{3m}{4\pi\rho} \right)^{2/3} \right] = k \left(\frac{1}{m} \right)^{1/3}$$

Therefore for the given two bodies.

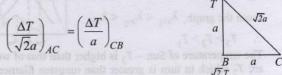
$$\frac{\Delta T_1 / \Delta t_1}{\Delta T_2 / \Delta t_2} = \left(\frac{m_2}{m_1}\right)^{1/3} = \left(\frac{1}{3}\right)^{1/3}$$

(b) According to question, temperature at $B = \sqrt{2}T >$ 22. temperatue at A = T so heat flow from B to A, A to C and C

For steady state condition, $\Delta Q/\Delta t$ is same.

Applying heat conduction formula $\frac{\Delta Q}{\Delta t} = \frac{k A \Delta T}{\ell}$

For sides AC and CB



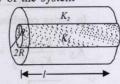


$$\Rightarrow \frac{T - T_c}{\sqrt{2}a} = \frac{T_c - \sqrt{2}T}{a} \Rightarrow T - T_c = \sqrt{2}T_c - 2T$$

$$\Rightarrow 3T = T_c(\sqrt{2} + 1) \Rightarrow \frac{T_c}{T} = \frac{3}{\sqrt{2} + 1}$$

(c) Let K = thermal conductivity of the system

Total transfer of heat per second through the combined system = Heat transfer per second from with material conductivity K_1 + Heat transfer per second from material with



thermal conductivity
$$K_2$$
.
$$\frac{KA\Delta T}{\ell} = \frac{K_1A_1\Delta T}{\ell} + \frac{K_2A_2\Delta T}{\ell}$$
 or,
$$K\pi (2R)^2 = K_1\pi R^2 + K_2\pi \left[(2R)^2 - R^2\right]$$
 or,
$$K = \frac{K_1 + 3K_2}{4}$$

(a) From principle of calorimetry, Heat lost by steam = Heat gained by (water + calorimeter) $mL + m \times c \times (100 - 80) = 1.12 \times c \times (80 - 15)$

Thermal Properties of Matter

$$m [540 + 1 \times 20] = 1.12 \times 1 \times 65$$

 $m = 0.13 \text{ kg}$

25. **(b)**
$$\frac{Q_2}{Q_1} = \frac{nC_v \Delta T}{nC_p \Delta T} = \frac{1}{\gamma} \Rightarrow Q_2 = \frac{Q_1}{\gamma}$$
$$\Rightarrow Q_2 = \frac{70}{1.4} = 50 \text{ cal}$$

26. (a) From,
$$V_{rms} = \sqrt{\frac{3RT}{m}}$$

$$M = \frac{3RT}{V_{rms}^2} = \frac{3 \times 8.314 \times 298}{1930 \times 1930} \times 1000 = 2g$$
Hence the gas is H.

27. (b) Under thermal equilibrium, $\theta_A - \theta_B = 36^{\circ}C$ (Given)

$$\theta_{C} = \frac{\frac{K_{A}}{\ell} \theta_{A} + \frac{K_{B}}{\ell} \theta_{B}}{\frac{K_{A}}{\ell} + \frac{K_{B}}{\ell}} \theta_{B}$$

$$\therefore \quad \theta_C = \frac{2\theta_A + \theta_B}{3} = \frac{2\theta_A + \theta_A - 36}{3} = \frac{3(\theta_A - 12)}{3}$$

$$\theta_A - \theta_C = 12^{\circ}C$$

Hence, the temperature difference across the layer $A = 12^{\circ}C$

28. (9) Heat radiated =
$$\sigma A T^4 = -ms \frac{dT}{dt}$$

So,
$$\int_{200}^{100} \frac{dT}{T^4} = \int_{0}^{t_1} k dt \implies \frac{1}{3T^3} \Big|_{200}^{100} = kt_1$$
or,
$$\frac{1}{3} \left(\frac{1}{100^3} - \frac{1}{200^3} \right) = kt_1 \qquad \dots (i)$$

Similarly,
$$\frac{1}{3T^3}\Big|_{200}^{50} = kt_2$$

$$\Rightarrow \frac{1}{3} \left(\frac{1}{50^3} - \frac{1}{200^3}\right) = kt_2 \qquad ...(ii)$$

$$\frac{t_2}{t_1} = \left(\frac{200^3 - 50^3}{200^3 - 100^3}\right) \frac{100^3}{50^3} = 9$$

29. (9) According to stefan's law, $P \propto T^4$ or $P = P_0 T^4$

$$\log_2 P = \log_2 P_0 + \log_2 T^4 \quad \therefore \log_2 \frac{P}{P_0} = 4\log_2 T$$
At T = 487°C = 760 K, $\log_2 \frac{P}{P_0} = 4\log_2 760 = 1 \dots (i)$
At T = 2767°C = 3040K, $\log_e \frac{\rho}{\rho_0} = 4\log_2 3040 = 4\log_2 (760 \times 4)$

$$= 4 \left[\log_2 760 + \log_2 2^2\right] = 4\log_2 760 + 8 = 1 + 8 = 9$$

(2) From (i) Stefan-Boltzmann law, $P = \sigma A T^4$ and (ii) Wein's displacement law = $\lambda_m \times T = \text{constant}$

$$\frac{P_A}{P_B} = \frac{A_A}{A_B} \frac{T_A^4}{T_B^4} = \frac{A_A}{A_B} \times \frac{\lambda_B^4}{\lambda_A^4}$$

$$\therefore \frac{\lambda_A}{\lambda_B} = \left[\frac{A_A}{A_B} \times \frac{P_B}{P_A}\right]^{\frac{1}{4}} = \left[\frac{R_A^2}{R_B^2} \times \frac{P_B}{P_A}\right]^{\frac{1}{4}} = \left[\frac{400 \times 400}{10^4}\right]^{\frac{1}{4}}$$
$$\therefore \frac{\lambda_A}{\lambda_B} = 2$$

31. (9) According to stefan-Boltzmann law,

Rate of total energy radiated by A Rate of total energy radiated by B

$$= \frac{\sigma T_1^4 (4\pi r_1^2)}{\sigma T_2^4 (4\pi r_2^2)} = \left(\frac{T_1}{T_2}\right)^4 \times \left(\frac{r_1}{r_2}\right)^2$$

$$= \left(\frac{\lambda_{m_2}}{\lambda_{m_1}}\right)^4 \left(\frac{r_1}{r_2}\right)^2 \quad \left[\because \frac{T_1}{T_2} = \frac{\lambda_{m_2}}{\lambda_{m_1}} \text{ by Wein 's law }\right]$$

$$= \left(\frac{1500}{500}\right)^4 \left(\frac{6}{18}\right)^2 = 9$$

(8) As there is no other heat exchange in this process, So Heat supplied = Heat used in converting m grams of ice from -5°C to 0°C + Heat used in converting 1 gram of ice at 0°C to water at 0°C

or,
$$420 = mc\Delta \theta + ml$$

$$\Rightarrow 420 = m \times \frac{2100}{1000} \times 5 + \frac{1 \times 3.36 \times 10^5}{1000}$$

⇒
$$420 = m \times 10.5 + 336$$
 ∴ $m = \frac{84}{10.5} = 8g$

33. (25000) Given, mass m = 1 kg, $T_i = -73^{\circ}\text{C} = 200\text{K}$ $T_f = 27^{\circ}\text{C} = 300\text{K}$

$$T_i = -73^{\circ}C = 200K$$

 $T_i = 27^{\circ}C = 300K$

$$Q = m.c. dT = 1kTdT$$

$$Q = \int_{200}^{300} 1.kT dT = k \int_{200}^{300} T dT$$

$$= \frac{k}{2} [T^2]_{200}^{300} = \frac{k}{2} [300^2 - 200^2] = 25000K$$

(270°C) Let C be the specific heat capacity of liquid and L be the latent heat of vapourisation.

From principle of calorimetry,

$$m_C S_C \Delta T = mC \Delta T + mL$$

or
$$m_C S_C (110 - 80) = 5C (80 - 30) + 5L$$

Where, $m_C = \text{mass of calorimeter}$

 $S_C = \text{sp. heat of calorimeter}$

Again, when 80g liquid is poured and equilibrium temperature is 50°C

$$m_C S_C(80-50) = 80C(50-30)$$

From eq. (i) & (ii) 1600 C = 250C + 5L

$$\therefore \frac{L}{C} = \frac{1350}{5} = 270^{\circ}C$$

Rate of heat flow will be same,

Rate of heat flow
$$\frac{dQ}{dt} = \frac{\text{temp. difference}}{\text{thermal resistance}} = \frac{1}{R} (T_2 - T_1)$$

SUN

where
$$R = \frac{L}{KA}$$

$$\frac{300 - 200}{R_1} = \frac{200 - 100}{R_2} \text{ or } R_1 = R_2$$

$$\frac{L_1}{K_1 A_1} = \frac{L_2}{K_2 A_2} \therefore \frac{K_1}{K_2} = \frac{A_2}{A_1} = \frac{\pi (2r)^2}{\pi r^2} = 4 [\because L_1 = L_2 = L]$$

36. Solar power received by 0.2 m^2 area $= (1400 \text{ W/m}^2) (0.2 \text{ m}^2) = 280 \text{ W}$ Mass of ice = 280 g = 0.280 kgHeat required to melt ice = mL

 $= (0.280) (3.3 \times 10^5) = 9.24 \times 10^4 \text{ J}$ If t is the time taken for the ice to melt,

$$\therefore (280)t = 9.24 \times 10^4 \text{ J} \qquad \left[\because P = \frac{E}{t}\right]$$

or,
$$t = \frac{9.24 \times 10^4}{280}$$
 s = 330 s = 5.5 min

37. Let t be the temperature of the interface after the steady state is reached.

The heat transferred per second through A

 $Q_1 = K_1 A (100 - t)$ The heat transferred per second through B

 $Q_2 = K_2 A (t-0)$ At steady state $K_1 A (100 - t) = K_2 A (t-0)$

- At steady state $K_1 A (100-t) = K_2 A (t-0)$ $\Rightarrow 300 (100-t) = 200 (t-0) \Rightarrow 300 - 3t = 2t \Rightarrow t = 60^{\circ} \text{ C}$
- 38. It takes t seconds for the substance to solidify (given). Therefore total heat released in t seconds = $P \times t = mL_{\text{fusion}}$

$$\therefore L_{\text{fusion}} = \frac{P \times t}{m}$$

39. Rate of heat transfer, $P = \frac{\text{temperature differ}}{\text{thermal resistance}}$

or
$$P = \frac{T}{t/4\pi R^2 K}$$
 $\left[\because \text{ Thermal resistance} = \frac{l(-t)}{KA}\right]$

 $\Rightarrow t = \frac{4\pi KTR^2}{P}$

i.e., thickness t should

not exceed,
$$\frac{4\pi KTR^2}{P}$$

40. Energy radiated per second

$$\sigma T^4 A = mc \frac{dT}{dt}$$

$$\Rightarrow dt = \frac{mcdT}{\sigma T^4 A} = \frac{\rho \times \frac{4}{3} \pi r^3 cdT}{\sigma T^4 \times 4\pi r^2} \left[\because m = \rho \times \frac{4}{3} \pi r^3 \text{ and } A = 4\pi r^2 \right]$$

$$\Rightarrow dt = \frac{\rho rc}{3\sigma} \frac{dT}{T^4}$$

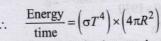
Integrating both sides

$$\int_0^t dt = \frac{\rho rc}{3\sigma} \int_{200}^{100} \frac{dT}{T^4} = \frac{\rho rc}{3\sigma} \left[-\frac{1}{3T^3} \right]_{200}^{100}$$

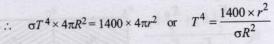
$$t = -\frac{\rho rc}{9\sigma} \left[\frac{1}{(100)^3} - \frac{1}{(200)^3} \right]$$
$$t = \frac{7\rho rc}{(72 \times 10^6)\sigma} \approx \frac{7\rho rc}{72 \times 10^6 (5.67 \times 10^{-8})} = 1.71\rho rc$$

Hence time required for the temperature of the sphere to drop to 100 k = 1.71 prc

41. According to Stefan's law, Energy radiated per unit time per unit area = σT^4



Energy received on earth per unit time = $1400 \times 4\pi r^2$



or
$$T^4 = \frac{1400 \times (1.5 \times 10^{11})^2}{(5.67 \times 10^{-8})(7 \times 10^8)^2}$$

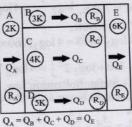
or
$$T^4 = \frac{14 \times 2.25 \times 10^{24}}{5.67 \times 49 \times 10^8}$$
 or $T = 5803$ K.

- water at 0° C = $mL = 100 \times 80 \times 4.2 = 33,600 \text{ J}$ The heat released by 300g of water at 25°C to change its temperature to 0° C = $mc\Delta T = 300 \times 4.2 \times 25 = 31,500 \text{ J}$ Hence complete ice will not melt, so the final temperature
- of the mixture will be 0°C.
 43. O → A material is in solid state only temperature is increasing. From A → B material is partly solid and partly liquid.

Since P is a point between A and B, therefore the material is partly solid and partly liquid.

- 44. (False) Energy radiated per second by the first sphere $E_1 = \varepsilon \sigma T^4 A = \varepsilon \sigma (4000)^4 \times 4\pi \times 11 \times 11 = 1024 \times \pi \times 10^{12} \times \varepsilon \sigma$ Energy radiated per second by the second sphere $E_2 = \varepsilon \sigma \times (2000)^4 \times 4\pi \times 4 \times 4 = 1024 \pi \times 10^{12} \times \varepsilon \sigma$ $\therefore E_1 = E_2$
- 45. (a, c, d)

According to question, heat Q flows only from left to right through the blocks. Hence heat flow through slab A and E are the same.



We know that thermal resistance $R = \frac{1}{KA}$

Let the width of slabs be W. Then

$$R_A = \frac{L}{2K(4L)W} = \frac{1}{8KW}, \ R_B = \frac{4L}{3K(LW)} = \frac{4}{3KW}$$

$$R_C = \frac{4L}{4K(2LW)} = \frac{1}{2KW}, \ R_D = \frac{4L}{5K(LW)} = \frac{4}{5KW}$$

$$R_E = \frac{L}{6K(4LW)} = \frac{1}{24KW}$$

Now, $\Delta T = QR$

Since the resistance to heat flow is least for slab E, the temperature difference across E is smallest.

$$Q_C = \frac{\Delta T_C}{R_C} = \frac{\Delta T_C}{1/2KW} = 2KW(\Delta T_C)$$

$$Q_B = \frac{\Delta T_B}{R_B} = \frac{\Delta T_C}{4/3KW} = \frac{3KW(\Delta T_C)}{4} \quad [\because \Delta T_B = \Delta T_C]$$

$$Q_D = \frac{\Delta T_D}{R_D} = \frac{\Delta T_C}{4/5KW} = \frac{5KW(\Delta T_c)}{4} \quad [\because \Delta T_D = \Delta T_C]$$

$$Q_B + Q_D = \frac{3KW(\Delta T_C)}{4} + \frac{5KW(\Delta T_C)}{4}$$

$$=\frac{8KW(\Delta T_C)}{4}=2KW(\Delta T_C)=Q_C$$

i.e., $Q_C = Q_R + Q_D$

- (d) Since the temperature of black body and chamber remains constant so energy emitted = energy absorbed by black body.
- 47. (a, b)

Energy emitted per second by body A and B are same.

$$\therefore = \varepsilon_A \sigma T_A^4 A = \varepsilon_B \sigma T_B^4 A$$

$$\therefore T_B = \left(\frac{\varepsilon_A}{\varepsilon_B}\right)^{1/4} \times T_A = 1934 \,\mathrm{K}$$

According to Wein's displacement law $\lambda_m \propto \frac{1}{T}$

$$(\lambda_m)_A T_A = (\lambda_m)_B T_B \Rightarrow \frac{(\lambda_m)_A}{(\lambda_m)_B} = \frac{T_B}{T_A} = \frac{5802}{1934}$$
 ... (i)

Since temperature of A is more therefore $(\lambda_m)_A$ is less $(\lambda_m)_B - (\lambda_m)_A = 1 \times 10^{-6} \,\mathrm{m}$ (given) Solving eq. (i) and (ii), we get $\lambda_B = 1.5 \times 10^{-6} \,\mathrm{m}.$

48. Heat lost by steam at 100°C to change to 100°C water $Q_1 = mL_{\text{vap}} = 0.05 \times 2268 \times 1000 = 1,13,400 \text{ J}$

Heat lost by 100°C water to change to 0°C water $Q_2 = MC\Delta T = 0.05 \times 4200 \times 100 = 21,000 \text{ J}$

Heat required by 0.45 kg of ice to change its temperature from 253 K to 273 K

 $Q_3 = m \times C_{\text{ice}} \times \Delta T = 0.45 \times 2100 \times 20 = 18,900 \text{ J}$

Heat required by 0.45 kg ice at 273 K to convert into 0.45 kg water at 273 K

 $Q_4 = mL_{\text{fusion}} = 0.45 \times 336 \times 1000 = 151,200 \,\text{J}$ From the above data it is clear that $Q_1 + Q_2 > Q_3$ but $Q_1 + Q_2 < Q_3 + Q_4$ so whole ice will not melt.

Therefore the final temperature will be 273 K or 0°C.

49. Let m be the mass of the container According to principle of calorimetry, heat lost = heat gain Heat lost by container = msdT or dQ = m(A + BT)dT

or
$$\int_0^Q dQ = m \int_{500}^{300} (A + BT) dT \Rightarrow Q = m \left[AT + \frac{BT^2}{2} \right]_{500}^{300}$$

or
$$Q = m \left[100T + \frac{2 \times 10^{-2}}{2} T^2 \right]_{500}^{300}$$

$$\Rightarrow Q = m \left[100(300 - 500) + \frac{(300)^2 \times (500)^2}{100} \right]$$

or Q = m [-2000 - 1600] calorie ...(i) or Q = -21600 m calorie

Heat gained by ice in melting = mL

 $Q_1 = 0.1 \times 80000 = 8000 \text{ cal}$

Heat gained by water of above ice = $ms\Delta T$

 $Q_2 = 0.1 \times 1000 \times 27 = 2700 \text{ cal}$

∴ Total heat gained = 8000 + 2700

$$Q_1 + Q_2 = 10700 \text{ cal}$$
 ...(i

 \therefore Heat lost = Heat gained or 21600 m = 10700

or
$$m = \frac{10700}{21600} = 0.495 \text{ kg}$$
 : Mass of container = 0.495 kg.

50. Heat lost by steam = Heat gained by water $m_s L_{\text{fus}} = m_w c \Delta T$

$$\Rightarrow m_s = \frac{m_w c \Delta T}{L_{fius}} = \frac{0.1 \times 4200 \times 66}{540 \times 10^3 \times 4.2} = 0.0122 \text{ kg}$$

51. The energy emitted per second by both spheres will be same as the temperature and surface area are same.

We know that $Q = mc\Delta T$

or,
$$\frac{dQ}{dt} = \frac{mcdT}{dt}$$

Since Q is same and c is also same (both of copper).

$$\therefore \frac{dQ}{dt} \propto \frac{1}{m}$$



Mass of hollow sphere is less so hollow sphere will cool faster.

52. Given 25% of the heat is absorbed by the obstacle. Therefore 75% heat is used in melting of lead. Initial temp. = 27°C

Metting point of lead = 327° C : $\Delta T = 327 - 27 = 300^{\circ}$ C

$$(0.75) \times \frac{1}{2} M v^2 = Mc \Delta T + ML$$

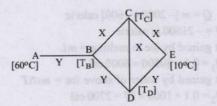
$$(0.75) \times \frac{1}{2}v^2 = (0.03 \times 300 + 6) \times 4.2$$

v = 12.96 m/s

- 53. (i) In region AB, heat is absorbed by the material at a constant temperature and is used in phase changing from solid to liquid.
 - (ii) In region CD heat is absorbed by the material at a constant temperature and is used in phase changing from liquid to gas.

Latent heat of vaporisation = 2 (latent heat of fusion)

- (iii) The slope *DE* indicates that the temperature of the solid begins to rise.
- (iv) The slope of OA > slope of BC. This indicates that there is a rise in specific heat.
- **54.** Given: Thermal conductivity $K_X = 0.92$ cal/sec-cm-°C $K_Y = 0.46$ cal/sec-cm-°C



From figure

$$\frac{K_Y A (60 - T_B)}{\ell} = \frac{K_X A (T_B - 10)}{2\ell} + \frac{K_Y A (T_B - 10)}{2\ell}$$

Solving the above equation, we get $T_B = 30^{\circ}$ C No heat will flow through CD

As C is a point at the middle of BE therefore temperature at

$$C = \frac{T_B + T_E}{2} = \frac{30^{\circ}C + 10^{\circ}C}{2} = 20^{\circ}C$$

Similarly temperature at D is also 20°C.

图_

Topic-3: Miscellaneous (Mixed Concepts)
Problems

1. (d) Power,

$$P = \frac{dQ}{dt} = \frac{d}{dt}(mc)T = (H)\frac{dT}{dt} = (H)\frac{d}{dt}\left[T_0(1+\beta t^{1/4})\right]$$

$$P = (H)T_0 = \frac{\beta t^{-3/4}}{4}$$
 where $H =$ heat capacity

$$\therefore (H) = \frac{4Pt^{3/4}}{T_0\beta} \qquad \dots (i)$$

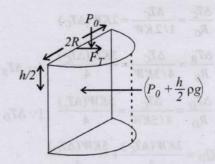
But
$$t^{1/4} = \frac{T(t) - T_0}{\beta T_0}$$

$$\therefore t^{3/4} = \frac{[T(t) - T_0]^3}{\beta^3 T_0^3} \qquad (ii)$$

From eq. (i) & (ii),

$$(H) = \frac{4P \left[T(t) - T_0 \right]^3}{T_0 \beta} = \frac{4P \left[T(t) - T_0 \right]^3}{\beta^4 T_0^4}$$

2. **(b)** In the first part the force is created due to pressure and in the second part the force is due to surface tension T.



The force is
$$\left[\left(P_0 + \frac{h \rho g}{2} \right) \times (2R \times h) \right] - 2RT$$

$$\therefore \quad \text{Force} = 2P_0Rh + R\rho gh^2 - 2RT$$

3. (d) By Stefan's law

For black body,
$$\frac{P}{A} = \sigma T^4$$

For hot bodies other than black body, $\frac{P}{A} = e\sigma A T^4$

Now, when such hot body is kept inside a perfectly black body, the total thermal radiation is sum of emitted radiations (in open) and the part of incident radiations reflected from the walls of the perfectly black body. This will give black body radiations, hence the total radiation emitted by the body will be $P = 1.0 \text{ e}GAT^4$.

4. (c) For a given mass of gas at constant pressure,

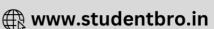
$$\frac{V}{T}$$
 = Constant $\therefore \frac{V + \Delta V}{T + \Delta T} = \frac{V}{T}$

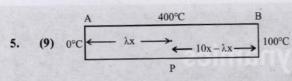
or $VT + V\Delta T = VT + T\Delta V$ or $V\Delta T = T\Delta V$

or
$$\frac{1}{T} = \frac{\Delta V}{V \Delta T}$$
 or $\frac{1}{T} = \delta$ or $\delta T = 1$

The equation represents a rectangular hyperbola of the form $xy = c^2$ depicted by graph (c).







For heat flow from P to 0

$$\frac{dQ_1}{dt} = L_f \frac{dm_1}{dt} = \frac{KA\,400}{\lambda x} \qquad \dots (i)$$

$$\frac{dQ_2}{dt} = L_{vap} \frac{dm_2}{dt} = \frac{KA\,300}{10x - \lambda x} \dots \text{(ii)} \left[\text{Given } \frac{dm_1}{dt} = \frac{dm_2}{dt} \right]$$

Dividing eq. (i) by (ii) and solving we get $\lambda = 9$

(8.33)

Rate of loss of heat,

$$\frac{dQ}{dt} = \sigma eA(T^4 - T_0^4) \qquad ...(i)$$

$$\Rightarrow \frac{dQ}{Adt} = e\sigma(T_0 + \Delta T)^4 - T_0^4 = \sigma T_0^4 \left[\left(1 + \frac{\Delta T}{T_0} \right)^4 - 1 \right]$$

$$= e\sigma T_0^4 \left[\left(1 + 4\frac{\Delta T}{T_0} \right) - 1 \right]$$

$$\frac{dQ}{Adt} = \sigma e T_0^3 \cdot 4\Delta T \qquad ...(ii)$$

Now from eq. (1)
$$ms \frac{dT}{dt} = \sigma eA(T^4 - T_0^4) \quad [\because Q = ms\Delta T]$$

$$\Rightarrow \frac{dT}{dt} = \frac{\sigma eA}{ms} [(T_0 + \Delta T)^4 - T_0^4]$$

$$= \frac{\sigma e A}{m s} T_0^4 \times \left[\left(1 + \frac{\Delta T}{T_0} \right)^4 - 1 \right]$$

$$\frac{dT}{dt} = \frac{\sigma eA}{ms} T_0^4 \cdot 4\Delta T \Rightarrow \frac{dT}{dt} = K\Delta T;$$

$$\left(K = \frac{4\sigma e A T_0^3}{ms}$$
Constant for Newton's law of cooling

$$\Rightarrow 4\sigma eAT_0^3 = \frac{K}{A}(ms)$$

$$\frac{dQ}{Adt} = e\sigma T_0^3 \cdot 4\Delta T$$

Since, rate of loss of heat = heat received per second $[K \times ms = 4200 \times 10^{-3}]$ $700 = (K/A) \text{ (ms) } \Delta T$

$$\Rightarrow \Delta T = \frac{700 \times A}{K \times ms} = \frac{700 \times 5 \times 10^{-2}}{10^{-3} \times 4200} = \frac{50}{6} = \frac{25}{3}$$

$$\Delta T = 8.33$$

(c) Energy radiated by the body = $\sigma A(T^4 - T_0^4)t$

[For a black body e = 1]

$$= \sigma A[(T_0 + 10)^4 - T_0^4]t$$

$$= \sigma A T_0^4 \left[\left(1 + \frac{10}{T_0} \right)^4 - 1 \right] t$$

$$= \sigma A T_0^4 \left[\frac{40}{T_0} \right] \times t = 460 \times 1 \times \frac{40}{300} \times 1 = 61.33J$$

$$P = \frac{\text{Energy radiated}}{\text{time}} = \sigma A T^4 - \sigma A T_0^4$$

$$\therefore \left| \frac{dp}{dT_0} \right| = \sigma A (4T_0^3) \quad \therefore \left| dp \right| = \sigma A (4T_0^3) dT_0$$

$$|\Delta P| = 4\sigma A T_0^3$$

Here as human body is not a black body. So option (a) and (b) are incorrect.

Energy radiated & A where A is the surface area of the body. Hence option (c) is correct.